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ABSTRACT

This tutorial presents a geometric proof done by sketch to compose a square or equilateral triangle from four small boards. It then converts the procedure into a series of 12 drawing steps with Geometer's Sketchpad. The complete Geometer's Sketchpad file of this demonstration is located at: http://poncelet.math.nthu.edu.tw/chuan/dissect/2dud.html. (ASK)

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Tutorial on Geometer's Sketchpad--Dudeney's Decomposition

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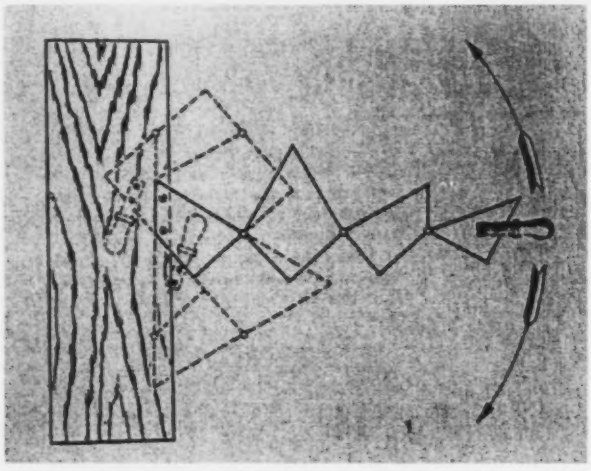
This is the first paragraph of the first section entitled "Triangles, Squares and Games" in the famous book "Mathematical Snapshots" written by H. Steinhaus:

From these four small boards (1) we can compose a square or an equilateral triangle, according as we turn the handle up or down. The proof is given by sketch (2).

Shown below are the accompanying figures in the book:



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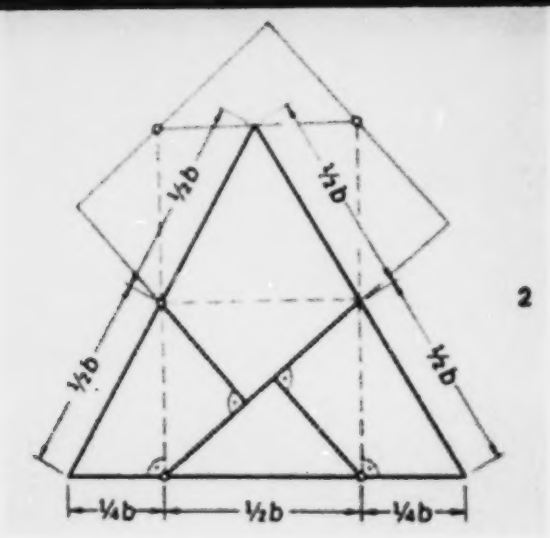

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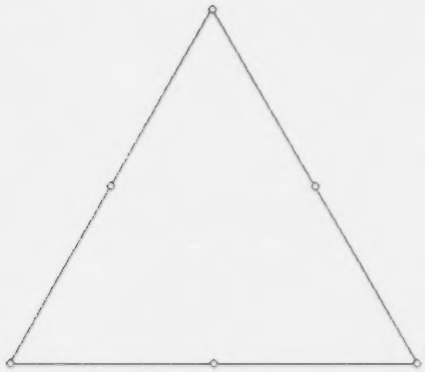
Question: **Is it possible to give a mathematically correct proof from sketch (2)?**

Steinhaus acknowledged that the idea of the decomposition was taken from p.27 of H.E. Dudeney's "Amusements in Mathematics." As we checked against the original source, there was no mention of this particular decomposition in the book at all! A figure included in Martin Gardner's book "More Mathematical Puzzles and Diversions", however, does enlighten us on the decomposition that Dudeney invented. We now convert the procedure into a series of drawing steps with Geometer's Sketchpad:

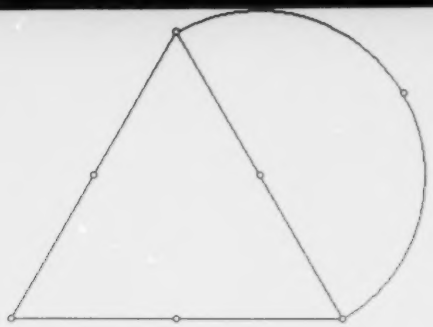
Step 1.

Construct the equilateral triangle together with the three midpoints.



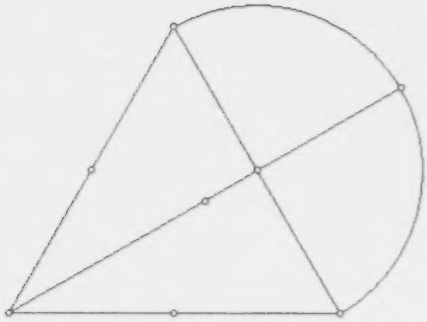
Step 2.

Construct an outward semi-circle by taking one side as diameter.



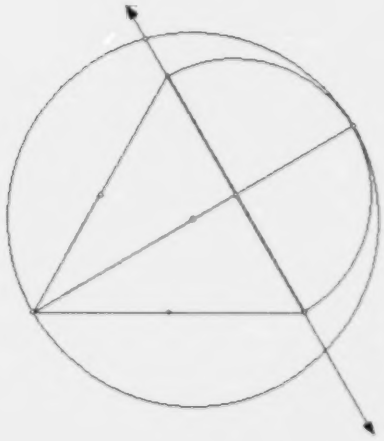
Step 3.

Draw the axis of symmetry.



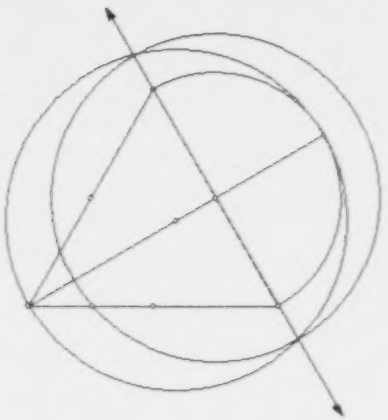
Step 4.

Draw a circle by taking the axis as diameter and find its intercept with the extension of the side of triangle. This way, the length of one side of the required square is found.



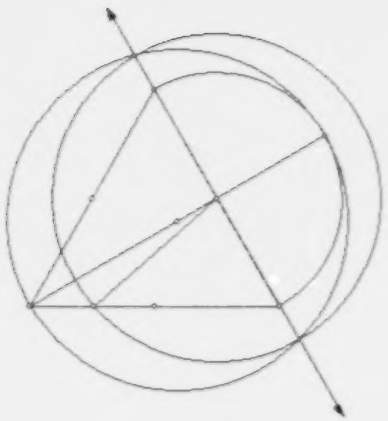
Step 5.

The position of the second vertex of the square is located.



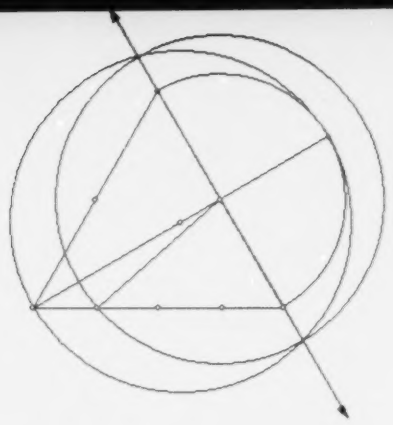
Step 6.

This is one side of the square.



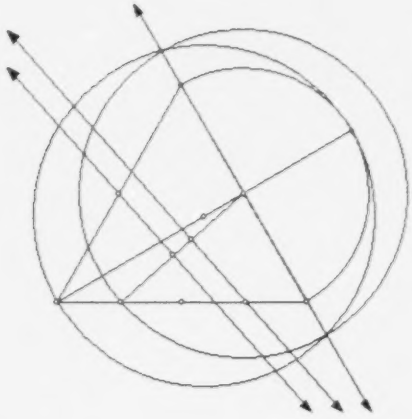
Step 7.

Locate a point with a distance one-half the length of the equilateral triangle from the point found in Step 6.



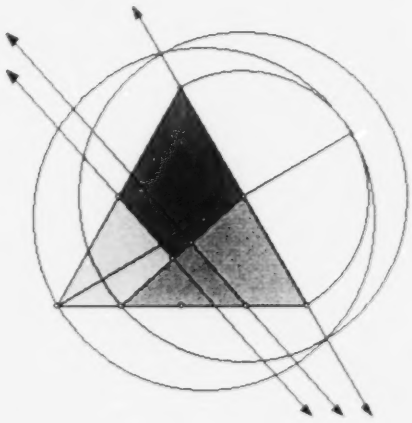
Step 8.

Drop perpendiculars to the last segment. The decomposition is now complete.



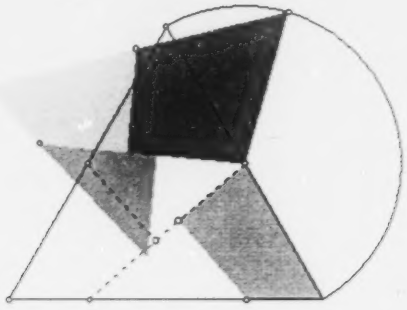
Step 9.

Fill the interiors of the four regions with distinct colors.



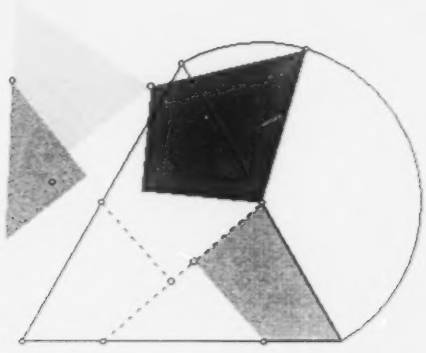
Step 10.

In order to re-assemble the four pieces into a square, place an arbitrary point on the semi-circle. The transformation from triangle to square is to be performed by moving this arbitrary point along the semi-circle. This is the first of three intermediate steps in the construction: rotate the three pieces with respect to the midpoint.



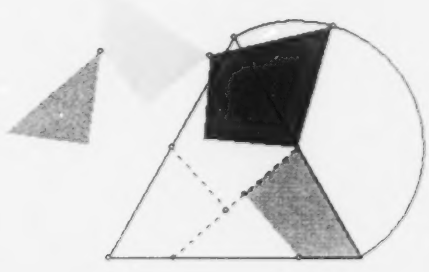
Step 11.

The second intermediate step: rotate the two pieces with respect to the second midpoint.

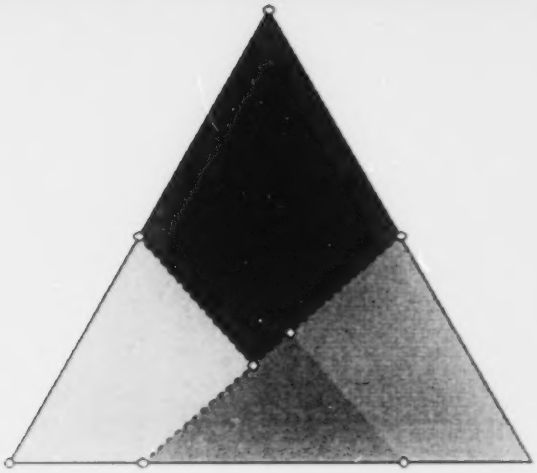


Step 12.

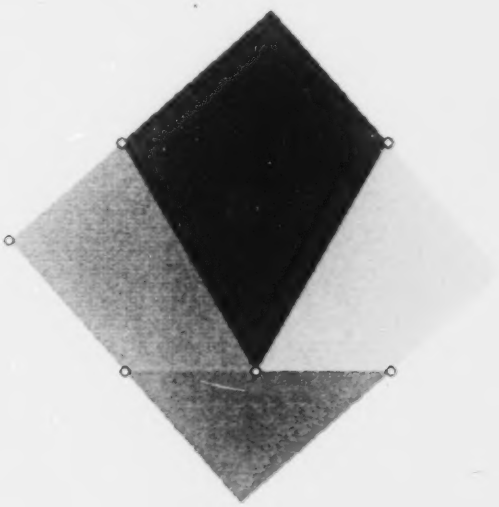
The third intermediate step: rotate the last piece with respect to the remaining midpoint.



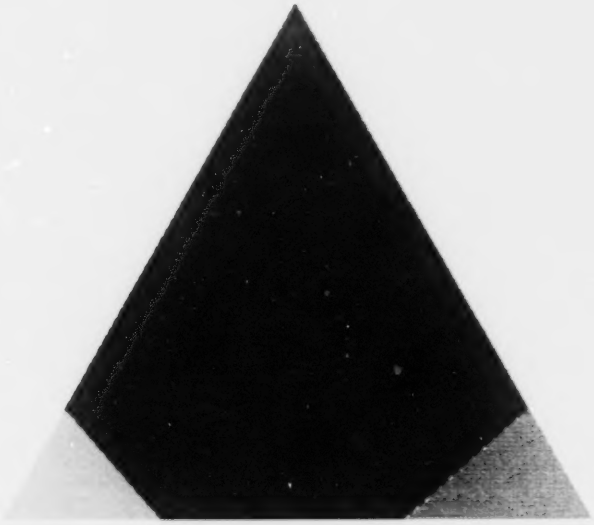
This way we may perform a continuous transformation by turing the decomposition of the triangle from



into the decomposition of the square.



Using the software we find the ratio BH/BL = 0.255 approximately.



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References

1. H.M. Cundy and A.P. Rollett, Mathematical Models, Tarquin, p. 24.
2. H.E. Dudeney's, Amusements in Mathematics, Dover (1970).
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4. Martin Gardner, More Mathematical Puzzles and Diversions, p. 26.
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